# 03C_Fracture Toughness

## The overarching question:

how do we study fracture in a formal, and quantitative way.

•For example, elastic deformation is defined in terms of stress and strain in a uniaxial test.

•Plastic deformation is defined by the yield stress (more exactly by yield at 0.2% offset strain), in a uniaxial tensile test.

About Fracture:

-depends on the applied stress and the flaw size.

-The fracture criterion should generalize the influence of both on crack propagation. $K_I = \alpha\sigma\sqrt{c}$, where $\alpha$ is a shape factor, $\sigma$ is the applied tensile stress, and $c$ is the length scale of the "flaw".

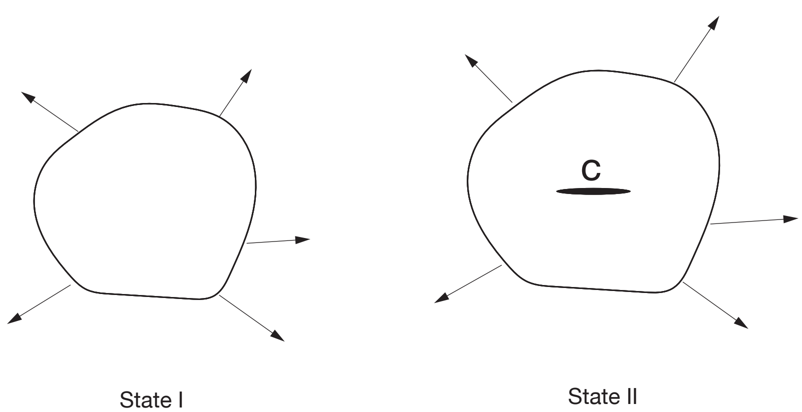
(i) Stress Intensity Factor ($K_I$): the equivalent of the applied tensile stress in a simple uniaxial experiment in crack propagation phenomenon.
(ii) Fracture Toughness ($K_{IC}$): The critical value of $K_I$ for crack propagation (if $K_I < K_{IC}$ then the state is subcritical)

# The Approach

The approach considers the work done to propagate the crack being provided by mechanical work in the system. The mechanical work can consist of two parts: the change in the potential energy (as in the constant load experiment) and the change in the stored elastic energy. The total mechanical work can be written as either the change in the elastic energy or one half of the change in potential energy.

# The Analysis

In general we can calculate the change in the elastic energy more conveniently than the change in the potential energy. For example consider the following



In general it is difficult to calculate the change in the potential energy. For example consider the two states shown above; it is almost impossible to ask for the change in the potential energy with and without the crack.

But, we can calculate the change in the elastic energy under a (local) uniform tensile stress.

Thermodynamic analysis consists of the work of fracture and the change in the mechanical energy.

Assume that the crack is penny-shaped with a diameter equal to $c$. And that the local tensile stress = $\sigma$. And we know that the elastic energy will depend on the Elastic Modulus, which we call $E$.

# Work of Fracture

The difference between State II and State I:

$$2\pi\left(\frac{c}{2}\right)^2 \gamma_F$$

# The Change in the Elastic Energy

Is given by

(elastic energy per unit volume remote from the crack, as in State I)= $\dfrac{\sigma^2}{2E}$ (1)

Equation one is derived from the elastic stress/strain line. The area when stressed to $\sigma$ is the area in the triangle which is equal to $\dfrac{\sigma\varepsilon}{2}$, and since $\varepsilon = \dfrac{\sigma}{E}$ which gives the result in Eq. (1)

One measures the load, $F$ as a function of displacement, $u$. So the elastic energy stored = $\dfrac{Fu}{2}$ units of J (N*m).

Cross section of the sample is $A$, and its length is $L$ then

$$\frac{Fu}{2} = \frac{1}{2}\frac{F}{A}A\frac{u}{L}L = \frac{1}{2}\sigma\varepsilon * (AL)$$

$$Volume = AL$$

$\dfrac{1}{AL}\dfrac{Fu}{2} = \dfrac{1}{2}\sigma\varepsilon$, Energy per unit volume.

Remember that stress Pa is also energy per unit volume because $\varepsilon$ is dimensionless.

Therefore, from Eq. (1) $\dfrac{\sigma^2}{2E}$ is the elastic energy stored per unit volume.

The question is what is the increase in the strain energy associated with crack $c$ in State II.

The simple result is:

That the extra (elastic) strain energy associated with the crack is given by

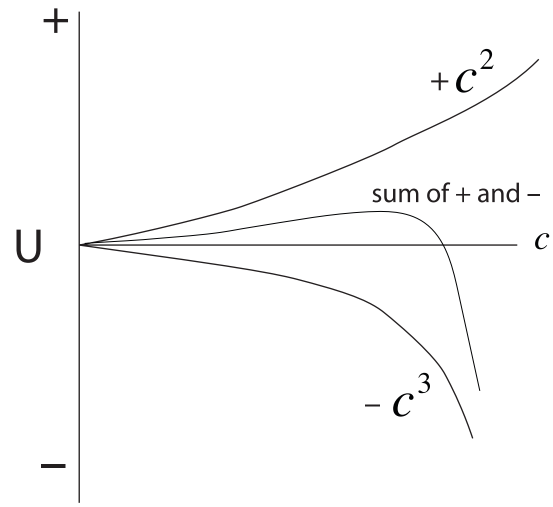
2*(effective volume of the crack)* $\dfrac{\sigma^2}{2E}$ (2)

Effective volume of the crack= $\dfrac{4}{3}\pi\left(\dfrac{c}{2}\right)^3$ (it is the volume of the sphere that can fit the crack within it. Note the extra factor of 2 in Eq. (2).

Therefore the total change in energy (State II -State I) is given by

$$U = +2\pi\left(\frac{c}{2}\right)^2 \gamma_F - 2 * \frac{4}{3}\pi\left(\frac{c}{2}\right)^3 \frac{\sigma^2}{2E} \qquad (3)$$

The two terms are shown in the schematic on the right. The $c^2$ term is positive and the $c^3$ term is negative. The sum of two has a maximum which is the critical condition for fracture. The maximum is given by




$$\frac{dU}{dc} = +2\pi\gamma_F \frac{2c}{4} - 2 * \frac{4}{3}\pi \frac{3c^2}{8}\frac{\sigma^2}{2E} = 0$$

$$2\gamma_F = \frac{\sigma^2 c}{E} \qquad (4)$$

$$K_{IC}^2 = \sigma^2 c \qquad (5)$$

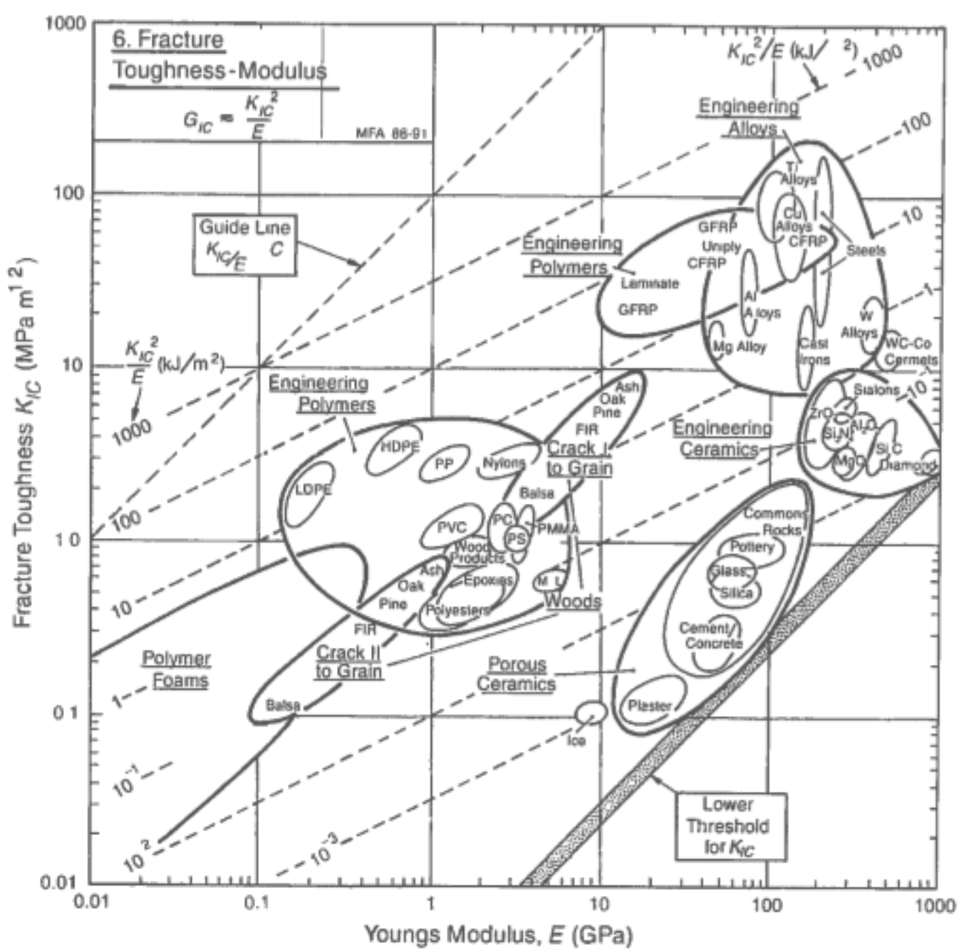
$$2\gamma_F = \frac{K_{IC}^2}{E} \qquad (6)$$

$K_{IC}$ is now, necessarily a material parameter because the other two in (6) are material parameters.

We can further say that for a sub-critical crack $K_I < K_{IC}$

And now we may say the $K_I$ specifies the "loading" of the crack (just as uniaxial stress specifies the loading oa f simple bar in a uniaxial test). It is called stress intensity.

The following figure shows a "map" of the fracture toughness vs. the Youngs Modulus, E for different classes of materials that includes polymers, ceramics, metals and graphite-fiber-polymer composites.




Notes:

•The dotted lines are for constant values of $2\gamma_F \; (= \frac{K_{IC}^2}{E})$